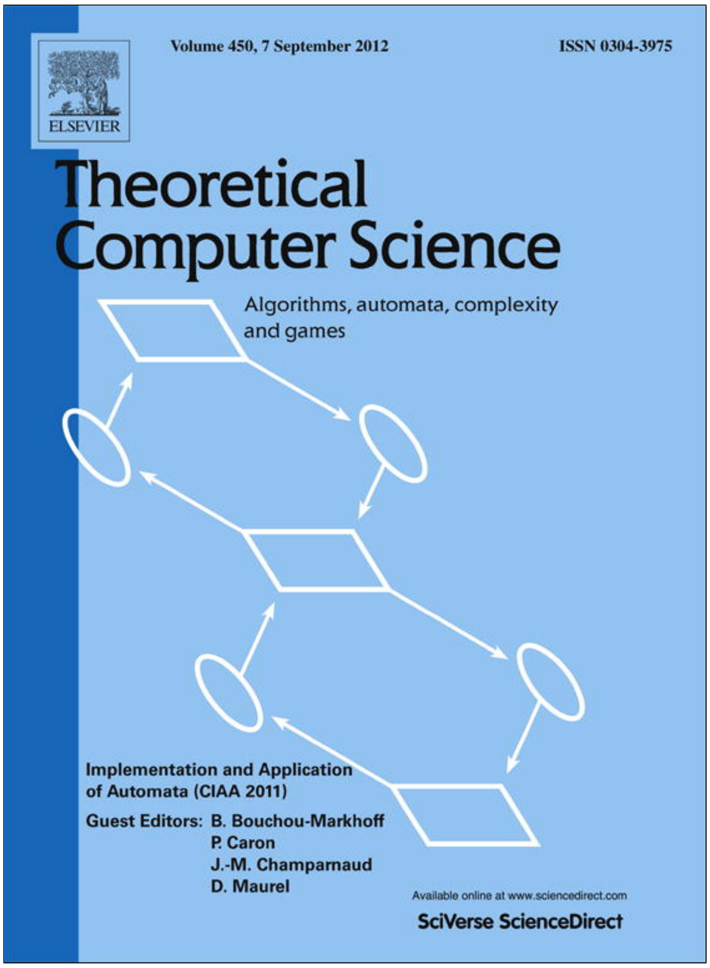
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Analysis of a cellular automaton model for car traffic with a junction☆

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A R T I C L E   I N F O

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A B S T R A C T

We propose a cellular automaton model that simulates traffic flow with a junction. We consider the form-one-lane rule and the merge-lane rule that decide which car moves ahead when two cars in two different lanes are in front of a junction. We simulate the proposed cellular automaton model for both rules, and generate fundamental diagrams and car distribution examples. Then, we analyze experimental results and demonstrate that the proposed model reflects the real world traffic flow with a junction according to the considered rules.

1. Introduction

Traffic jams are a big problem in most major cities throughout the world. Several researchers have attempted to reproduce real-world traffic flow as accurately as possible and to predict possible traffic jam conditions based on simulations. One approach is to use a cellular automaton (CA) for modeling traffic flow. CA models are intuitive and they can simulate a complex behavior using a set of simple rules. Wolfram [29] presented a basic one-dimensional CA model for highway traffic flow (the R184 rule). Nagel and Schreckenberg [15] designed a traffic simulation model using CAs, that is a variant of the R184 rule [29]. The Nagel and Schreckenberg model (in short, NaSch model) shows a transition from laminar traffic flow to start-stop-waves as the car density increases using Monte-Carlo simulations. Benjamin et al. [1] developed another model (BJH model), which is an extended version of the NaSch model with a slow-to-start rule. The BJH model considers the flawed behavior of drivers. The slow-to-start rule assumes that drivers sometimes become distracted after having been stuck in the queue of stopped cars and, thus, start with some delay. Clarridge and Salomaa [6] proposed a slow-to-stop rule and added the new rule into the BJH model. The slow-to-stop rule is based on the fact that when drivers see a traffic jam or congestion ahead, they start decelerating to avoid collision. Note that these models are designed for single-lane traffic using one-dimensional CAs. However, there is often more than a single lane with several junctions where two or more lanes join and this may cause heavy traffic congestion. See Fig. 1 for an example. This motivates us to examine traffic flow in a multi-lane system with a junction.

Benjamin et al. [1] examined the presence of a junction. They studied the effects of acceleration, disorder and slow-to-start behavior on the queue length at the entrance to the highway. Xiao et al. [31] analyzed a bridge traffic bottleneck based on the R184 model [29,30]. Researchers investigated two-lane or multi-lane traffic simulations using CAs and lane changing rules [16,17,28]. Moreover there are several studies on the simulation models for complex urban traffic [3,5,7,18,24,26].

We focus on traffic flow with a junction where two lanes join. We note that there are different rules for deciding which lane has a higher priority; namely, a car in a higher priority lane has the right to move ahead and a car in a lower priority lane must yield. For instance, when a branch lane (lower priority) is merged into a main lane (higher priority), a car from the

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Fig. 1. An example of a traffic jam around a junction where several lanes join.

branch lane must yield to a car in the main lane and may enter the main lane when its entry does not slow cars in the main lane. For different junction rules, we compute an optimal traffic density that maximizes the traffic flow without causing a traffic jam. We use two one-dimensional CA arrays with various parameters for traffic flow. We define the traffic flux to be the number of cars passing through the lane in a time step. We calculate the flux of a junction and the length of traffic congestion in front of a junction and simulate other possible cases with a junction.

2. CA-based traffic simulation models

A CA is a collection of cells on a grid that evolves through a number of discrete time steps according to a set of rules based on the states of neighboring cells [30]. The NaSch model [15] is the first nontrivial traffic simulation model based on CAs. There are several papers that analyze this model in detail [14,20,22,27,21] and modify the model for better simulations [8,11,12]. The NaSch model [15] is defined on a one-dimensional array with periodic boundary conditions. Each cell may either be occupied by a car or be empty. Each car has an integer velocity with values between zero and v_{max}. For an arbitrary configuration, one update of the system consists of four consecutive steps performed in parallel for all cars. These four steps are acceleration, slowing down, randomization and car motion.

The BJH model [1] is an extension of the NaSch model [15]. Benjamin et al. [1] noticed that drivers may start slowly when they pull away from being in a static queue of cars. This slow start can arise from a driver's loss of attention as a result of being stuck in the queue. The BJH model introduces p_{slow} to simulate the driver's behavior stochastically. p_{slow} is the probability of starting slowly from a static queue of cars. When the velocity of a car is 0 and the distance between the next car is long enough, this car stays at velocity 0 in this time step with probability p_{slow} and accelerates to 1 in the next time step. Conversely, this car may accelerate normally with probability $1 - p_{slow}$. This rule is called the slow-to-start rule.

Lastly, there is one more rule for more realistic traffic simulation: the slow-to-stop rule developed by Clarridge and Salomaa [6]. They observed that the cars following the previous models behave in an unrealistic fashion when approaching a traffic jam. If a car B ahead has velocity 0, then a car A may drive up to B at velocity v_{max} only to brake down to velocity 0 in one time step in the cell right behind B. Based on the observation, they suggested the addition of a slow-to-stop rule. This rule causes drivers to go slower when approaching jams since drivers would slow down beforehand where a small jam is visible from a distance. Clarridge and Salomaa [6] applied this rule to the BJH model. Their simulation results demonstrate that there are fewer long jams with many cars at a complete stop, and instead there are many slowdowns to avoid the complete stop, which is more realistic than the BJH model.

Several researchers have studied multi-lane models for highway traffic. Rickert et al. [19] designed a two-lane cellular automaton model based on the NaSch model and noticed that checking for extra space when drivers switch lanes is

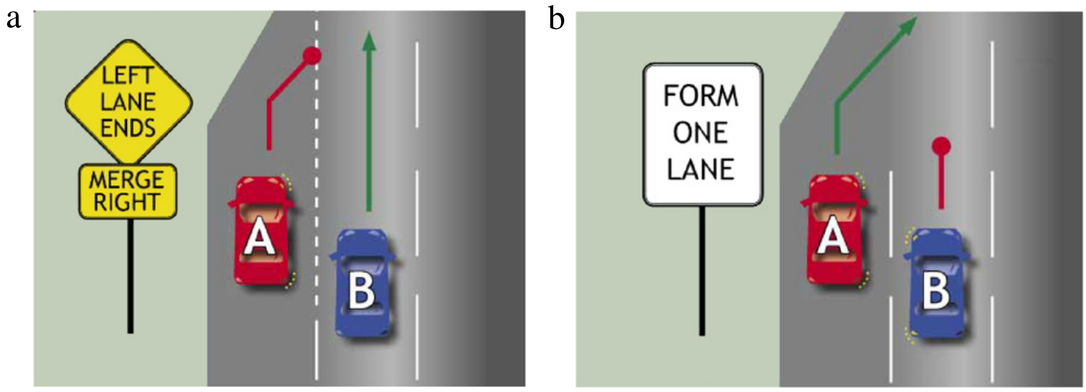

Fig. 2. (a) Merging traffic and (b) forming one lane. These two cases are the same in that two lanes join at a junction and become a single lane.

important in order to simulate the realistic behavior. Wagner et al. [28] proposed a set of lane changing rules for simulating multi-lane traffic, which accounts for the fact that the passing lane becomes more crowded than the one for slower cars if the flux is high enough. Knospe et al. [10] studied heterogeneous two-lane traffic and discussed the effect of slow cars in two-lane systems. Even a small number of slow cars cause both lanes to slow significantly. Nagel et al. [17] summarized different approaches to lane changing and their results and proposed a general scheme according to which realistic lane changing rules can be developed.

Esser and Schreckenberg [9] designed a complete simulation tool for urban traffic based on the NasCh model. Various roads and crossings are modeled as combinations of basic elements. They also considered parking capacities as well as circulations of public transports. There are several studies related to urban traffic [3,5,7,18,24,26,23].

Emmerich and Rank [8] studied various modifications to the NasCh model aiming at a quantitative improvement of the fundamental diagram obtained. Their results show qualitative and quantitative coincidence of maximum flux with values taken from real traffic measurements. Knospe et al. [11] proposed an improved discrete model incorporating anticipation effects, reduced acceleration capabilities and an enhanced interaction horizon for braking. The modified model is able to reproduce the three phases (free-flow, synchronized, and stop-and-go) observed in real-world traffic. Makowiec and Miklaszewski [12] modified the NasCh model by the assumption that each car has an individual velocity limit. Mallikarjuna and Rao [13] studied the heterogeneous traffic in which the physical and mechanical characteristics of different vehicles vary widely which in turn leads to complex traffic behavior. They presented a detailed description of the methodology used in developing the basic structure of an improved discrete CA model used to generate different traffic scenarios. Bham and Benekohal [2] developed a cell-based traffic simulation model (CELLSIM) for the simulation of high volume traffic at the regional level. This model has been validated at the macroscopic and microscopic levels using two sets of field data. Chakroborry and Maurya [4] proposed a framework for evaluations such as time-headway distribution, acceleration noise, stability in car-following situations and so on. They also presented the results of the evaluation of six existing CA-based models.

We present a CA-based simulation model for car traffic when two lanes join at a junction. There are two rules for cars entering the single lane from each of the two lanes. We develop two rules for modeling the traffic and analyze the properties with examples.

3. Proposed models

We propose new CA transition rules for traffic simulation with a highway junction where two lanes become a single lane.

When two lanes join at a junction, there are two types of rules: merging and forming one lane [25]. Merging traffic occurs when a lane is ending and a driver is required to cross a broken or dotted line to merge with other traffic. In this case, the driver who is about to cross the broken line must yield to traffic in close proximity in another lane regardless of which car is in front. This case is shown in Fig. 2(a); car *A* must yield to car *B*. The form-one-lane rule requires a driver to yield to a car in another lane if that car is in front of the driver's car when the lanes merge. Hence, the driver in front has the right of way. This case is shown in Fig. 2(b); car *B* must yield to car *A*.

3.1. Form-one-lane rule model

We first consider the form-one-lane. Under this rule, we give the same priority to all lanes. This implies that all cars on the road have the same priority regardless of which lane they are in.

In the form-one-lane rule, when there are two cars near a junction, the car that is in front of the car on the other lane goes first. We can adopt this rule to the velocity rule of the BJH model and the slow-to-stop model. In the BJH model, the next velocity of the car is determined based on the current velocity, the maximal velocity and the distance between the next car. The slow-to-stop model uses one more variable, the velocity of the next car, to determine the next velocity.

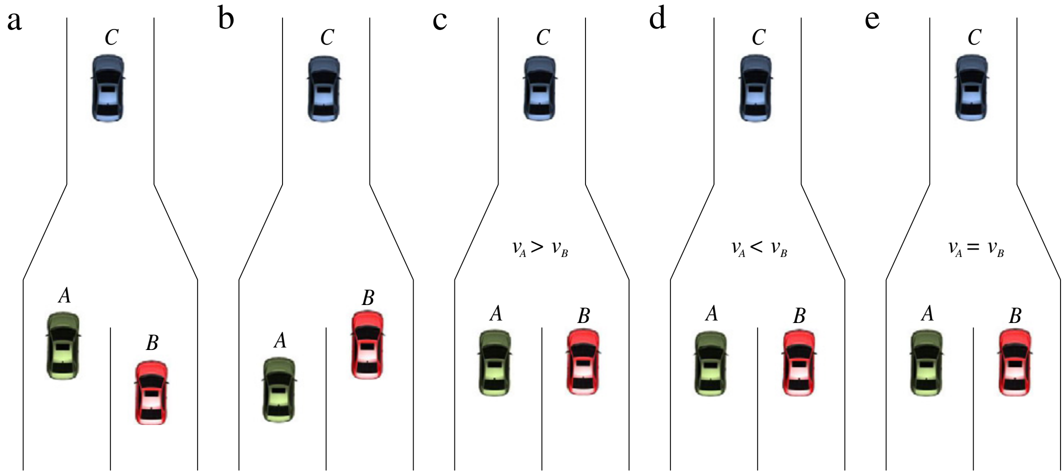

Fig. 3. Five possible cases with a junction where two lanes join. A and B are in the same position in (c), (d) and (e).

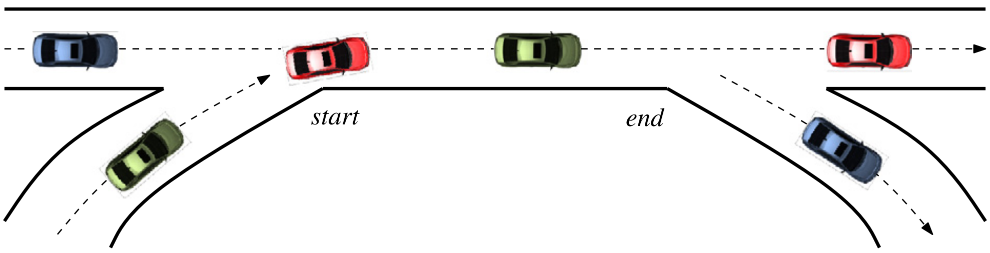

Fig. 4. Our model simulates the traffic flow where two lanes join at a junction and later split into two lanes. Let *start* and *end* be the beginning and the end of the joint single lane, respectively.

There are five possible cases with a junction as illustrated in Fig. 3. In trivial cases, there are two cars approaching a junction in each lane and they are at different positions. Let L_1 and L_2 be the two lanes that join and make a junction and L_3 be the joined lane after the junction. Let A, B and C be the closest cars to the junction in L_1, L_2 and L_3, respectively. In Fig. 3(a), A goes first because A is in front of B. Let $next(A)$ denote the next car of A. Then $C = next(A)$ and $A = next(B)$ in Fig. 3(a). Similarly B goes first, and $C = next(B)$ and $B = next(A)$ in Fig. 3(b).

Assume that $A = next(B)$. This implies that we can denote the distance of B as $pos(A) - pos(B)$, where $pos(A)$ is the index of the cell occupied by A. However, sometimes two cars in front of a junction can be in the same position; namely, $pos(A) - pos(B) = 0$. In the real world, under this condition, the faster car goes first. This is quite reasonable since the faster car is likely to reach the junction earlier than the other car. These cases are depicted in Fig. 3(c) and (d). In (c), A goes first and $A = next(B)$ since A is faster than B.

We also consider another case: two cars that are the same distance from a junction and drivers are traveling at the same velocity as depicted in Fig. 3(e). We can assume that one of the drivers would decrease their car's velocity because otherwise the two cars would crash into each other. We introduce a simple rule to avoid a crash. We randomly select a car and reduce the velocity of the selected car with probability p_{follow} to mimic real world behavior. Since the two lanes have the same priority, we set $p_{follow} \leftarrow 0.5$. Now the two cars have different velocities and thus we can follow one of the two cases in Fig. 3(c) and (d). We say that two cars A, B are in the same position if $pos(A) = pos(B)$.

Fig. 4 is an illustration of the considered traffic flow. Two lanes join and become a single lane and later split again. We assume that each car moves in the same lane before and after the merged single lane. For instance, if a car entered the junction from L_1, then it exists the merged single lane to L_1 as well. We use *start* and *end* to denote the beginning and the end of the joint single lane and $\mathcal{N}(start, i)$ to denote the car nearest to the junction in the direction of traffic in lane i. Recall that d is the distance to the next car, v is the velocity of the current car, v_{next} is the velocity of the next car, p_{slow} is the probability for the slow-to-start rule and p_{fault} is the probability that the car slows down. The cars A and B are the cars described in Fig. 3. We define the form-one-lane rule as follows:

1. In the joint single lane: If $start \le pos(A) < end$, then d is the distance from A to the nearest car ahead of A.
2. Too far from a junction: If $pos(B) + v_B + 1 < start$, then d is the distance between the car that is in front and in the single lane.
3. Closest to a junction: If $pos(\mathcal{N}(start, 1)) = pos(A) > pos(\mathcal{N}(start, 2))$, then d is the distance from A to the nearest car C ahead of A and we set v_{next} as the velocity of C (Fig. 3(a)).
4. Behind another car on the other lane: If $pos(\mathcal{N}(start, 1)) = pos(A) < pos(\mathcal{N}(start, 2))$, then $d \leftarrow pos(\mathcal{N}(start, 2)) - pos(A)$ and $v_{next} \leftarrow v(\mathcal{N}(start, 2))$ (Fig. 3(b)).

5. On the same position: When $pos(\mathcal{N}(start, 1)) = pos(A) = pos(\mathcal{N}(start, 2))$.
   (a) If $v_A > v(\mathcal{N}(start, 2))$, then d is the distance between the car that is in front and in the single lane (Fig. 3(c)) .
   (b) If $v_A < v(\mathcal{N}(start, 2))$, then we set $d \leftarrow 0$ in order to stop the car and follow the car in the other lane. In this case, we do not need to set v_{next} since this car stops here regardless of the velocity of the next car (Fig. 3(d)).
   (c) If $v_A = v(\mathcal{N}(start, 2))$, then with probability p_{follow} d is the distance between the next car in the single lane, v_{next} is the velocity of that car, and the velocity of the car in the other lane decreases ($v(\mathcal{N}(start, 2)) \leftarrow v(\mathcal{N}(start, 2)) - 1$) (Fig. 3(e)).
6. Otherwise: The car is not affected by the junction. Let C be the car that is ahead of A. Then d is the distance from A to C and v_{next} is the velocity of C.

The form-one-lane rule determines d and v_{next} for each car closest to the junction in each lane. We do not apply the rule to the cars that cannot approach the junction position ($start$) in the next time step. However, it is impossible to determine the values of all cars in parallel with these rules. When two cars are in the same position and have the same velocity, we reduce the velocity of one car with probability p_{follow}. Then the two cars have different velocities and the slower car follows the faster car. If these rules are applied to all cars in parallel, then the velocity of two cars can be reduced at the same time and this causes both cars to stop. Thus, for two cars with the same velocity, we avoid this problem by applying the form-one-lane rule to each car one-by-one.

The following is the velocity rule for calculating the velocities of all cars:

1. Slow-to-start: If $v = 0$ and $d > 1$, then the car accelerates on this step or stays there and accelerates on the next step.
2. Deceleration (when the next car is near): If $d \leq v$ and either $v < v_{next}$ or $v \leq 2$, then the next car is either very close or going at a faster speed, and we prevent a collision by setting $v \leftarrow d - 1$, but do not slow down more than is necessary. Otherwise, if $d \leq v$, $v \geq v_{next}$, and $v > 2$ we set $v \leftarrow min(d-1, v-2)$ in order to possibly further decelerate, since the car ahead is slower or moving at the same speed and the velocity of the current car is substantial.
3. Deceleration (when the next car is far): If $v < d \leq 2v$, then if $v \geq v_{next} + 4$, decelerate by 2 ($v \leftarrow v - 2$). Otherwise, if $v_{next} + 2 \leq v \leq v_{next} + 3$ then decelerate by one ($v \leftarrow v - 1$).
4. Acceleration: If the speed has not been modified yet and $v < v_{max}$ and $d > v + 1$, then velocity increases ($v \leftarrow v + 1$).
5. Randomization: If $v > 0$, with probability p_{fault}, decelerate by 1 ($v \leftarrow v - 1$).
6. Car motion: The car advances v cells.

Now we design a form-one-lane model by using the form-one-lane rule and the velocity rule together. The main concern of our model is determining the next car when two cars are near a junction, especially when they are the same distance away from the junction. Note that when two cars are the same distance away from the junction, by our rules, the faster car has d as the distance to the car that is in front of the two cars and v_{next} as the velocity of that car, while the other car has d as 0. By the deceleration rule, if d is 0, then the velocity of the car becomes -1. However, since the domain of velocity is from 0 to v_{max}, we set velocity as 0. It follows that when two cars are the same distance away from the junction, one car stops there and thus two cars cannot advance at the same time.

The FormOneLane procedure determines d and v_{next} of A in the form-one-lane rule.

3.2. Merge-lane rule model

We examine the proposed CA transition rules for merging traffic. As shown in Fig. 2(a), even if car A is in front of car B, A must give way to B in merging traffic since the right lane has higher priority than the left lane. We design the merge-lane rule that reflects different priorities between two lanes where the two lanes join at a junction. Assume that the lane with a higher priority is L_1 and the other lane is L_2. When a car in L_1 is about to reach a junction, this car can advance to the merged single lane without considering cars in L_2. On the other hand, a car in L_2 cannot advance to the single lane since this car should pay attention to the status of L_1. Namely, if a car closest to the junction in L_1 can advance to the merged single lane in the next time step, then the car in L_2 should stay in front of the junction until there is no car in L_1 that can advance to the single lane in the next time step.

It follows that we use two different rules for cars in different lanes for determining d and v_{next} for each car closest to the junction in each lane. For the cars in L_1 (higher priority lane), we adopt the velocity rule described in Section 3.1 since the driver of a car in L_1 does not need to consider cars in L_2 (lower priority lane). On the other hand, the merge-lane rule imposes more restrictions on the cars in L_2.

1. In the joint single lane: If $start \leq pos(B) < end$, then d is the distance from B to the nearest car ahead of B.
2. Too far from a junction: If $pos(B) + v_B + 1 < start$, then d is the distance between the car that is in front and in the single lane.
3. Closest to a junction: When $pos(\mathcal{N}(start, 2)) = pos(B) > pos(\mathcal{N}(start, 1))$ (Fig. 3(b)).
   (a) If $pos(\mathcal{N}(start, 1)) + v(\mathcal{N}(start, 1)) + 1 \geq start$, then d is the distance between the car and the beginning of the joint single lane and v_{next} is $v(\mathcal{N}(start, 1))$.
   (b) Otherwise, d is the distance between the car that is in front and in the single lane.
4. Behind another car in the other lane: If $pos(\mathcal{N}(start, 2)) = pos(B) < pos(\mathcal{N}(start, 1))$, then $d \leftarrow start - pos(B)$ and $v_{next} \leftarrow v(\mathcal{N}(start, 1))$ (Fig. 3(a)).
5. In the same position: If $pos(\mathcal{N}(start, 1)) = pos(A) = pos(\mathcal{N}(start, 2))$, then $d \leftarrow start - pos(B)$ and $v_{next} \leftarrow v(\mathcal{N}(start, 1))$ (Fig. 3(c), (d), (e)).

Procedure FromOneLane

```
for all A in a lane do
  Set B as the nearest car to a junction in the other lane
  if start ≤ pos(A) < end then
    Set d as the distance between the car that is in front and v_next as its velocity
  else if the car A is closest to a junction in that lane then
    if pos(A) + end + 1 < start then
      Set d as the distance between the car that is in front and v_next as its velocity
    else
      if pos(A) > pos(B) then
        Set d as the distance between the car that is in front and v_next as its velocity
      else if pos(A) = pos(B) then
        if v_A > v_B then
          Set d as the distance between the car that is in front and v_next as its velocity
        else if v_A = v_B then
          if p_follow > 0.5 then
            Set the car that is in front and in the the single lane as next(A)
            Decrement v_B by one
          else
            d ← 0 and v_next ← v_B
            Decrement v_A by one
          end if
        else
          d ← 0 and v_next ← v_B
        end if
      else
        d ← pos(B) − pos(A) and v_next ← v_B
      end if
    end if
  else
    Set d as the distance between the car that is in front and v_next as its velocity
  end if
end for
```

6. Otherwise: The car is not affected by the junction. Let C be the car that is ahead of B. Then d is the distance from A to C and v_{next} is the velocity of C.

The merge-lane rule applies for the car B in Fig. 3 that is in the lane with a lower priority. When B is the closest car to a junction and not too far from the junction, we consider the possibility that car A reaches *start* in the next time step. Since cars can accelerate their velocities at most by one in a time step, if A accelerates and then is able to reach *start*, we assign $start - pos(B)$ to d of B and the velocity of A to v_{next}. Otherwise, A keeps advancing to the joint single lane without any consideration for B. When B is behind A or B is in the same position as A, unlike in the form-one-lane rule, we apply the same rule described above for both cases. In short, cars in L_2 can advance to the joint single lane when cars in L_1 cannot enter the single lane in the next time step and A should be ahead of B at that time.

In the real world, when two lanes merge at a junction with different priority, drivers in the lower priority lane (L_2) should pay attention to the other lane. Then, even though drivers in L_2 are ahead of drivers in L_1, they should yield to drivers in L_1 if they are likely to enter the single lane soon. For a similar reason, when drivers in L_2 are behind drivers in L_1 and moving at a higher velocity, they should yield since L_1 has a higher priority than L_2. The merge-lane rule reflects these properties in order to accurately reproduce real world behavior.

The MergeLane procedure determines d and v_{next} of B in L_2 by the merge-lane rule. Note that A denotes the nearest car to a junction in L_1.

4. Experiments and analysis

4.1. Single-lane model and the proposed model

We simulate the proposed model for traffic with a junction and compare the simulation results with an example of the slow-to-stop model developed by Clarridge and Salomaa [6]. Fig. 5 is an example of two simulations.

Procedure MergeLane

for all B in a lane L_2 (with a lower priority) **do**
  Set A as the nearest car in the other lane
  if $start \le pos(B) < end$ **then**
    Set d as the distance between the car that is in front and v_{next} as its velocity
  else if car B is closest to a junction in that lane **then**
    if $pos(B) + v_B + 1 < start$ **then**
      Set d as the distance between the car that is in front and v_{next} as its velocity
    else
      if $pos(B) > pos(A)$ **then**
        if $pos(B) + v_B + 1 \ge start$ **then**
          $d \leftarrow start - pos(B)$ and $v_{next} \leftarrow v_A$
        else
          Set d as the distance between the car that is in front and v_{next} as its velocity
        end if
      else if $pos(B) = pos(A)$ **then**
        $d \leftarrow start - pos(B)$ and $v_{next} \leftarrow v_A$
      else
        $d \leftarrow start - pos(B)$ and $v_{next} \leftarrow v_A$
      end if
    end if
  else
    Set d as the distance between the car that is in front and v_{next} as its velocity
  end if
end for

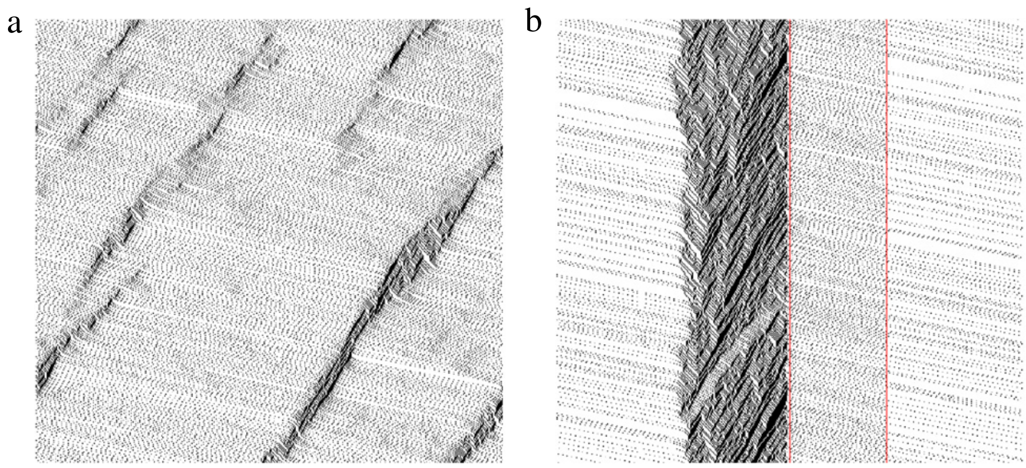

Fig. 5. The two pictures depict the distribution of cars over 500 consecutive time steps in a lane. (a) is an example of the slow-to-stop model proposed by Clarridge and Salomaa [6] and (b) is an example of the proposed simulation model. The other lane for (b) shows a similar distribution. The two vertical lines denote the beginning and the end of a single lane. Cars move from left to right.

First, we define some parameters for simulation: v_{max} is the maximum velocity of the cars. The cars on cells can move to the right at most v_{max} cells for each time step. p_{fault} and p_{slow} are the probabilities for the disorder rule and the slow-to-start rule, respectively.

In the simulation shown in Fig. 5, we use the following parameter values: $v_{max} = 5, p_{fault} = 0.1, p_{slow} = 0.5$ and $\rho = 0.15$. We calculate the traffic density ρ by the ratio of the number of cars to the number of cells. In our model, there are two lanes at first that become a single lane at a certain point. We simulate with the same density (0.15) for both lanes.

Notice that the start-stop-waves (traffic jams) in Fig. 5(a) often appear in real-world traffic. These jams move backwards slowly as the time passes and occur randomly. Therefore, it is difficult to predict where traffic jams occur and to estimate the length of traffic jams. On the other hand, the location of traffic jams in our model are consistent because of the presence of the junction. Fig. 6 shows the distribution of cars near a junction. The numbers represent the velocities of cars and the symbol | denotes the beginning of the merged single lane. The position right after | is the start position of the merged lane, namely, *start*.

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. . . . . . . . 4 . . . . . 2 . . 0 0 . . 1 . . 1 0 . 1 . 1|. 2 . . . 3 . . . . . . . . 5 . . . . . . . . . 5 . . . . .
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Fig. 6. This example precisely shows the distribution of cars near a junction in a lane. The other lane shows a similar distribution. The numbers represent the velocities of cars, each horizontal row is the car configuration for each time step and the vertical axis is the flow of time. Two lanes are simulated with densities that are both 0.08, where $p_{fault} = 0.1$ and $p_{slow} = 0.5$.

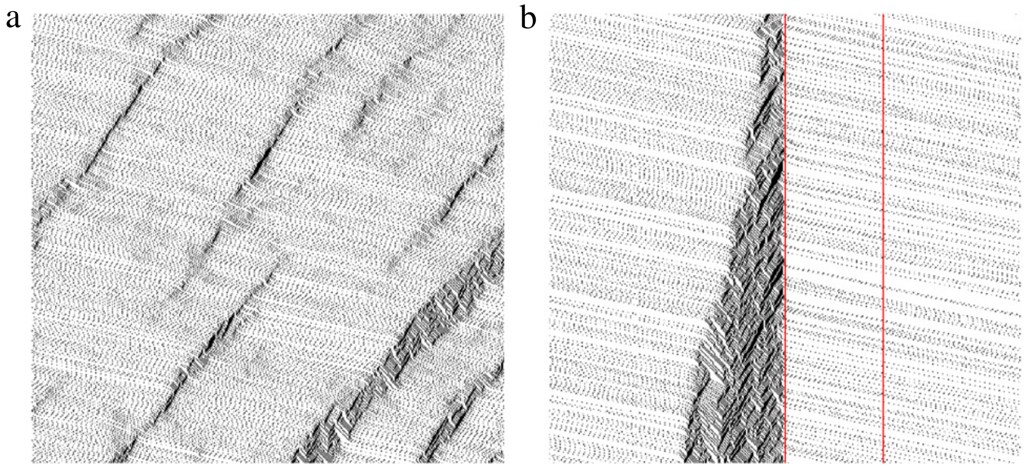

Fig. 7. The two pictures depict the distribution of cars over 500 consecutive time steps. We use randomly generated roads to add the cars to a stretch of road. Figure (a) is an example of the single lane model and figure (b) is an example of the form-one-lane model.

As shown in Fig. 5(b), most start-stop-waves occur in front of the traffic junction. This is quite similar to real-world traffic flow; traffic jams often occur in front of traffic junctions when the number of lanes decreases. The lengths of traffic jams are almost the same when we simulate with a fixed number of cars. This is because our simulation uses a circular road. In our simulation, there is a fixed number of cars on a fixed number of cells. The influx, which is the number of cars coming into the start of the lane in each time step, is the same as the outflux of the lane after a junction. This means that the flux of a junction repeatedly goes into the start of the lane along the circular road. Thus, the influx and outflux are the same and this why the lengths of traffic jams remain constant over time.

However, roads are typically not circular in reality. Furthermore, if the influx is larger than the maximal flux that the junction can process, then the number of cars before the junction would increase. On circular roads, it is impossible to simulate this. Clarridge and Salomaa [6] addressed this problem with an alternative method using Bernoulli process arrivals. This method adds new cars to a stretch of road instead of using CAs with circular boundary conditions. We resolve this issue by generating a road with a desired flux and a fixed density of cars. For instance, when we make a road with density 0.15, the flux is maximal (about 0.52). If we make a road with density 0.07, then the influx of each lane is 0.34. The initial configuration in the simulation is to randomly deploy cars to preserve the traffic density value. Since all initial velocities are zero, we run the simulation 1,000 time steps with the velocity rule and obtain a steady state. In Fig. 7, we confirm that the traffic jam no longer shows periodic trends and the lengths of the traffic jams increase monotonically.

4.2. Comparisons between the merge-lane rule and the form-one-lane rule

We compare the two rules based on simulation results. The form-one-lane rule and the merge-lane rule decide which car move ahead when two cars in different lanes are about to enter the merged lane at a junction. The form-one-lane rule considers two merging lanes with the same priority and the merge-lane rule considers two merging lanes with different priorities.

Fig. 8 shows car distributions of two lanes based on the form-one-lane rule. The two distributions are very similar to each other since we apply the same rule and priority to both lanes. On the other hand, Fig. 9 shows car distributions of two a b

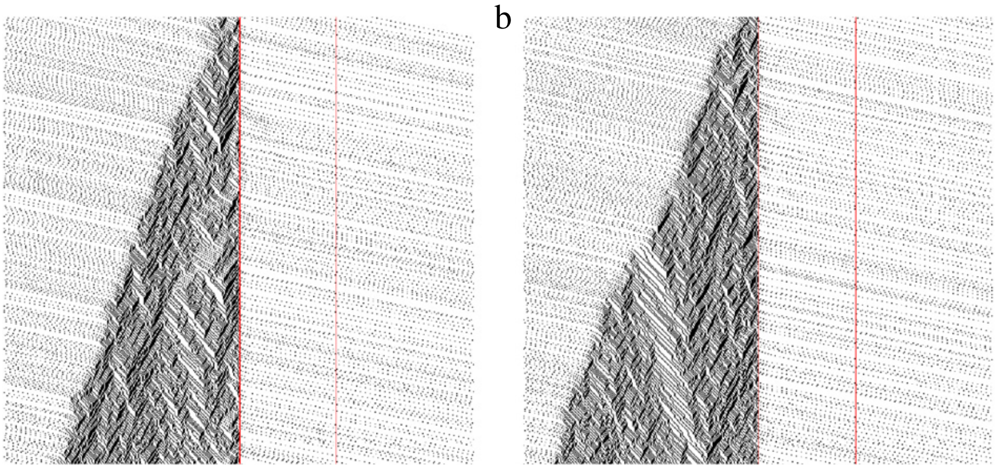

Fig. 8. The two pictures depict car distributions of two merging lanes under the form-one-lane rule. We set the density for both L_1 and L_2 as 0.8, $p_{fault} = 0.1$ and $p_{slow} = 0.5$. The two vertical lines denote the beginning and the end of the merged single lane. Cars move from left to right.

a b

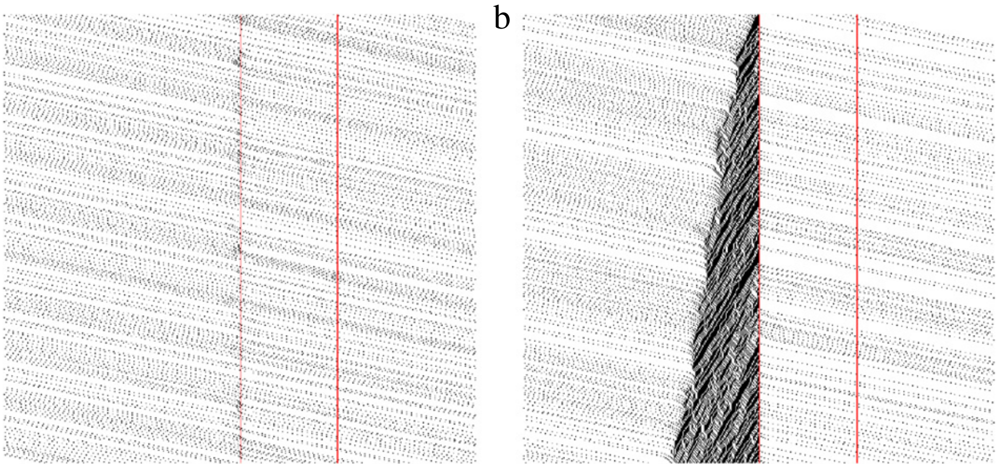

Fig. 9. The two pictures depict car distributions of two merging lanes under the merge-lane rule. (a) is the higher priority lane L_1 and (b) is the lower priority lane L_2. We set the density for both L_1 and L_2 as 0.06, $p_{fault} = 0.1$ and $p_{slow} = 0.5$. The two vertical lines denote the beginning and the end of the merged single lane. Cars move from left to right.

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Fig. 10. This example shows the distribution of cars in L_1 near a junction under the merge-lane rule. The numbers are car velocities. We set the density of L_1 as 0.1 and the density of L_2 as 0.02. The v_{max} values are 5 and 3, respectively. We use $p_{fault} = 0.1$ and $p_{slow} = 0.5$.

lanes with different priorities based on the merge-lane rule. Fig. 9(a) is the car distribution of the higher priority lane L_1. This distribution looks as if there is no junction since there are only very small regions of high density. These small jams only appear directly before the junction since L_1 has higher priority and thus all cars in L move forward without slowdown. Fig. 9(b) is a car distribution for L_2. Note that there are severe traffic jams before the junction. As explained in Section 3.2, cars in L_2 should often yield to cars in L_1 before entering the merged single lane. This gives rise to traffic jams.

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. . . . . . . . . 3 . . . . . . . . . . . . . . . . . . 0 0|. . . . . 2 . . 2 . . . 3 . . . . . . 3 . . . . 3 . 0 . . 1
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. . . . . . 3 . . . . . . . . . . . 3 . . . . . . . . . 0 0|. 2 . . . 3 . . . . . . . 3 . . . 3 . 1 . . . . 1 . . 1 . .
. . . . . . . . . 3 . . . . . . . . . . 2 . . . . . . . 0 0|. . . . 3 . . . 3 . . . . . . . 3 . 1 . 1 . . . . 1 . . . 2
. . . . . . . . . . . . 3 . . . . . . . . . . 3 . . . . 0 0|2 . . . . . . 3 . . . 3 . . . . . 1 . 1 . 1 . . . . . 2 . .
. . . . . . . . . . . . . . . 3 . . . . . . . . . 2 . . 0 0|. . . 3 . . . . . 2 . . 1 . . . . . 1 . 1 . . 2 . . . . . 2
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. . . . . . . . . . . . . . . . . . . . . . . . . 1 . 0 0 0|. 3 . . . 3 . 1 . . . . . 2 . . . 2 . . 2 . . . . . . 3 . .
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Fig. 11. This example shows a distribution of cars in L_2 near a junction under the merge-lane rule. The numbers are car velocities. We use the same settings in Fig. 10.

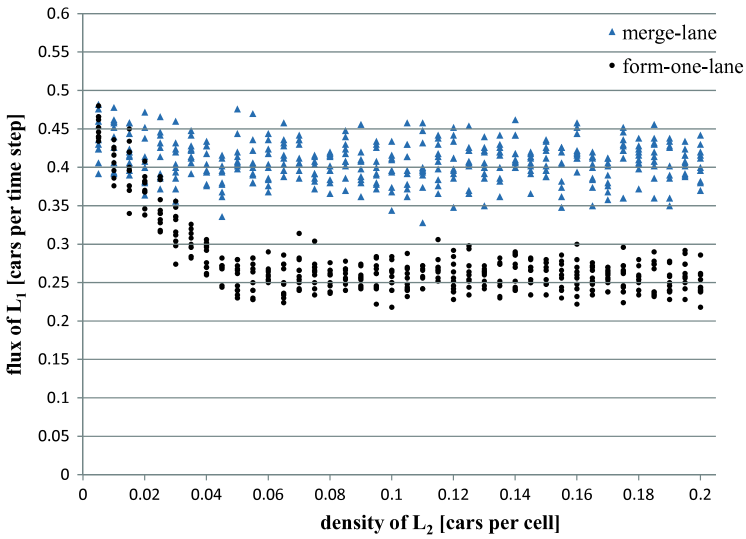

Fig. 12. The fundamental diagram of our model when we apply both rules. This diagram shows the relationship between the flux of L_1 and the varying density of L_2. We simulate with two lanes, where lane L_1 has a constant density of 0.15 and the other lane L_2 has a varying density. In the simulation, we increase the density of L_2 by 0.005 in each step.

Figs. 10 and 11 show magnified car distributions near a junction under the merge-lane rule for L_1 (higher priority) and L_2 (lower priority), respectively. Note that cars in Fig. 10 move without a significant traffic jam near the junction, whereas many cars in Fig. 11 are blocked from entering the merged single lane even if there are enough spaces ahead for the cars to move forward since they must yield to the cars in the other lane.

4.3. *Traffic flux with a junction*

Since the construction of new roads or traffic facilities is expensive, it is better to predict traffic flow before construction. This is especially important when attaching a new road to an existing road in order to prevent potential traffic jams resulting from the new junction. The proposed model can be used to estimate traffic flow when building a new road and creating a junction. The traffic flux is defined as the number of cars passing through the lane in a time step. This is an important factor to consider when adding new roads or building traffic facilities since high flux implies efficient traffic flow. Thus, it is best to maximize the traffic flux.

We examine the case when a lane L_1 is at maximal flux and the density of the other lane L_2 varies. We fix the density of L_1 at 0.15[1] and vary the density of L_2[2]. The fundamental diagram of this simulation depicted in Fig. 12 shows the result (black disks) under the form-one-lane rule and the result (blue triangles) under the merge-lane rule. The form-one-lane rule simulation shows a linear decrease until the density of L_2 becomes 0.06. After 0.05, the flux of L_1 stabilizes around 0.26. This is because the length of the queue of static cars in front of the traffic junction no longer affects the minimal flux of L_1.

1 Clarridge and Salomaa [6] showed that the flux is maximal when the density of the traffic is around 0.15.

2 A different density for L_1 shows a similar fundamental diagram.

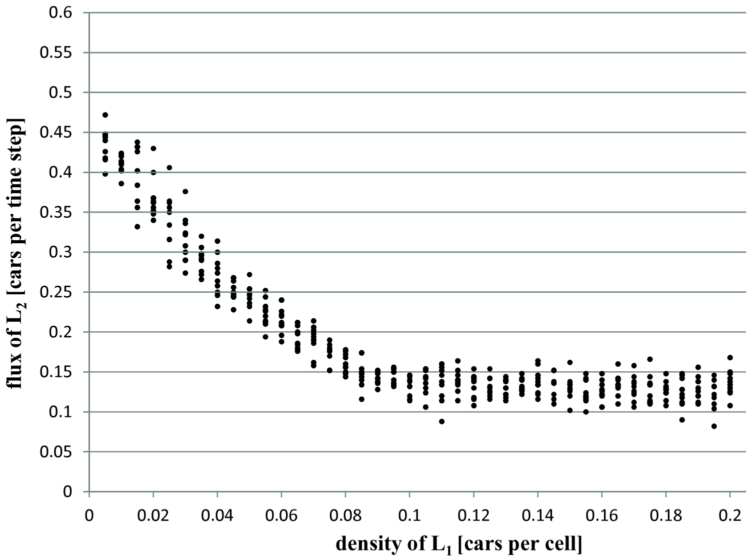

Fig. 13. The fundamental diagram of our model when we apply the merge-lane rule. This diagram shows the relationship between the flux of L_2 and the varying density of L_1. We simulate with two lanes, where a lane L_2 has a constant density of 0.15 and the other lane L_1 has a varying density. In the simulation, we increase the density of L_1 by 0.005 in each step.

Based on this observation, we can estimate the worst-case time complexity to pass the junction. On the other hand, for the merge-lane rule simulation, there is almost no change in the flux of L_1 when the density of L_2 varies.

We obtain interesting results by observing another case where L_2 is at maximal flux (the density is 0.15) and the density of L_1 varies. Fig. 13 is a fundamental diagram for this case and shows a linear decrease in flux until the density of L_1 becomes 0.09. Thereafter, the flux of L_2 stabilizes around 0.13. This result indicates that the lane with a lower priority in the merge-lane model has a minimal flux (0.13) that is not affected by the density of the other lane.

4.4. Two lanes with different roles

Two lanes with different roles can sometimes join at a junction and become a single lane. For instance, multi-lane highways consist of overtaking lanes, bus-only lanes[3] and normal lanes in many countries. We simulate the situation where two lanes have different roles, one of which is a bus-only lane and the other is a normal lane, and join at a junction.

We can adjust v_{max} and p_{slow} to make the two lanes different. v_{max} is the maximum velocity given for each car in the simulation and p_{slow} is the probability that the car stays there when its velocity is 0. We can use a higher p_{slow} and a lower v_{max} to mimic the behavior of a bus.[4]

The simulation result for two lanes with different roles is depicted in Fig. 14. Even though the two lanes have the same density, the car distributions are very different. For the normal lane, the length of the traffic jam increases in front of a junction as time passes. On the other hand, the bus-only lane has only small traffic jams. This is an important observation since when we apply the form-one-lane rule to two lanes under the same conditions, the car distributions of the two lanes are very similar. Now, we note that by applying a lower v_{max} and a higher p_{slow} for a lane, we can alleviate the traffic jam caused by a junction.

Fig. 15 shows the simulation result when we apply the bus-only lane condition to L_2 while the other conditions are the same as that in Fig. 9. We can alleviate the development of a traffic jam caused by a traffic junction by imposing the bus-only lane condition to L_2.

4.5. Traffic jam length

Since the traffic jams in our simulations often occur at a traffic junction, it is possible to compute the length of traffic jams and make use of them more easily than general start-stop waves. In Fig. 5(b), for instance, we can estimate the rough length of the jams. With fixed traffic densities the traffic jam lengths are similar in our simulation. Assuming real-world traffic flow in similar to that in our simulations, then we can use these simulation results for predicting the length of actual traffic jams at a junction. However, since there is no general method of measurement for traffic jams, we design a new rule for the simulation. When we decide whether or not traffic is jammed, we focus on the partial density of the traffic. If part of

[3] Bus-only lane is a lane restricted to buses and generally used to speed up public transport that would be otherwise held up by traffic congestion. The reader can find example cities using bus-only lanes from http://en.wikipedia.org/wiki/Bus_lane.

[4] A bus accelerates slowly and has a lower top speed compared with normal vehicles.

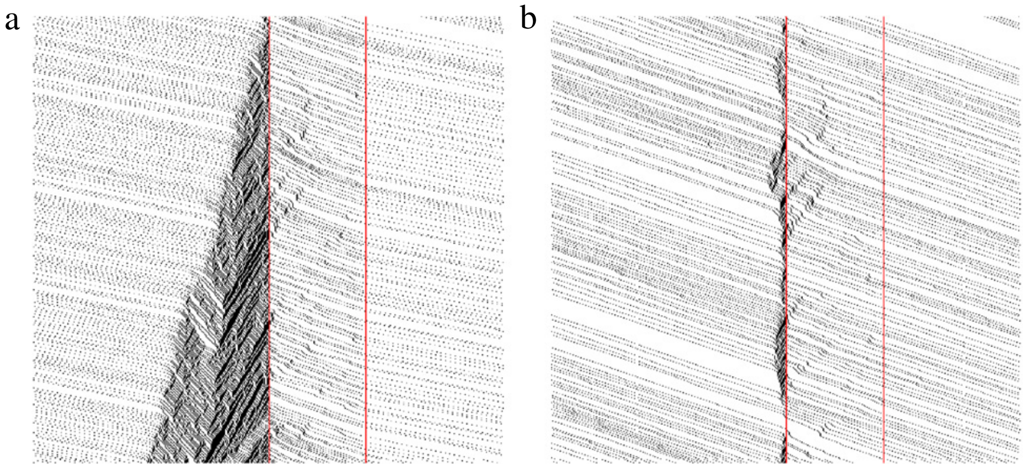

Fig. 14. These two pictures show car distributions where (a) is a normal lane and (b) is a bus-only lane. We apply the form-one-lane to simulate the traffic. It is simulated with two lanes whose densities are both 0.07, where $p_{fault} = 0.1$. v_{max} and p_{slow} are 5 and 0.5 for the normal lane and 3 and 0.7 for the bus-only lane, respectively.

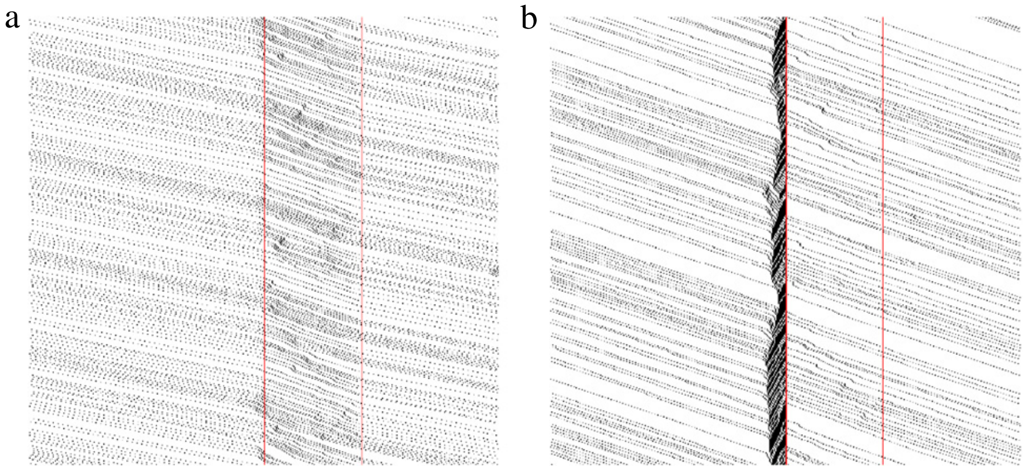

Fig. 15. This simulation is almost the same with that for Fig. 9. The only difference is that we apply the bus-only lane condition to L_2 ($v_{max} = 3$ and $p_{slow} = 0.7$).

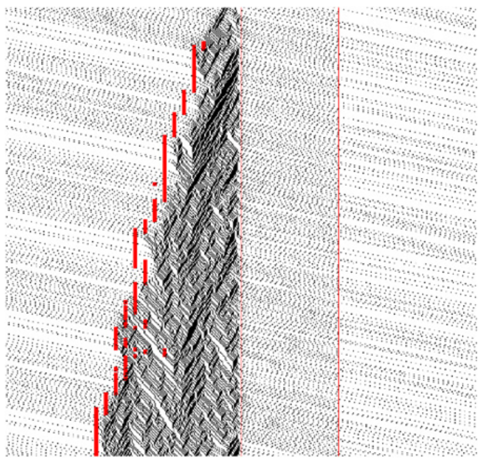

Fig. 16. Thick dots denote the point where the traffic jam begins. The standard density is 0.4.

the traffic is denser than the other parts, then we consider the denser part as the traffic jam. For the region from the starting point of the traffic to a junction, we measure the average density of the traffic. If the density is lower than the standard, then the starting point moves to the right, and stops at the point where the average density is higher than the standard density.

We indicate the starting point of the traffic jam as a thick dot in Fig. 16. We use a standard density of 0.4 since the maximal density is approximately 0.4 when a car is in a complete jam in our model. We observe that the length of the traffic jam increases linearly under these conditions.

4.6. Real-world scaling of the proposed model

When we apply the simulation results to real-world traffic flow, a quantitative comparison is needed. Since the unit in our simulation is abstract, we need some scaling between our model and real-world traffic. One scaling approach is to use the maximal velocity. Assume that the maximal velocity of real-world traffic is 100 km/h. The maximal velocity in our simulation is 5 cells per time step and, therefore, we can scale two values based on the following proportional equation:

$$\frac{5\text{ cells}}{1\text{ time step}} = \frac{100\text{ km}}{1\text{ hour}} = \frac{100000\text{ m}}{3600\text{ seconds}}.$$

Nagel and Schreckenberg [15] claimed that, in a complete jam, each car occupies about 7.5 m of space, which is used as the length of one cell. Similarly, if we regard the size of a cell as 7 m, then the time scale is 1.26 time steps to one second. Based on this scaling, we can calculate either the time to escape from the jam or the estimated arrival time. If a car in our simulation takes n time steps to escape from the jam and enters the single lane, we can simply convert n time steps to $\frac{n}{1.26}$ seconds in real time. In a similar way, we can predict the length of traffic jams in real-world traffic.

The second method is the comparative method. We can simulate traffic when there is a junction with various parameters using our model. When the traffic is in the standard condition and has a traffic jam with a length of 10 km in reality, we can simulate the condition with our model and obtain a traffic jam with a length of 50 cells. Then, we can simulate other parameters such as various amounts of flux and obtain the expected length of the traffic jam. If the length of the traffic jam is, say, 100 cells in the simulation, it becomes 20 km.

5. Conclusions

When two lanes join, there is always a junction. We have proposed a CA-based traffic simulation model with a form-one-lane rule and a merge-lane rule for simulating traffic flow with a junction. We have considered five possible cases near a junction based on the BJH model [1] and the slow-to-stop model [6], and demonstrated that the proposed model can simulate traffic near a junction realistically.

With some diagrams and examples, we have analyzed the experimental results and examined the length of traffic jams when the traffic density varies in addition to the flux of the traffic with a junction. We also have compared two rules from various angles. The flux of the lane stabilizes when the density of a lane is greater than a certain threshold. Based on this observation, we can predict the worst-case arrival time when the density of the other lane varies. We have suggested two approaches for using the proposed model to analyze real-world traffic. In the future, we need to compare empirical traffic data with our simulation results. From the real-world data and the simulation results, we can adjust parameters and adopt a new scale ratio to facilitate more precise predictions.

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